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On Viscosity Solutions of Path Dependent PDEs

Abstract: It is well known that Markovian BSDEs (resp. 2BSDEs) provides a probabilistic representation for the viscosity solution of a semi-linear (resp. fully nonlinear) parabolic PDE. In these lectures we shall introduce a type of parabolic Path Dependent PDEs (PPDEs, for short) and propose a notion of its viscosity solutions, and thus extends the above results to non-Markovian (or path dependent) cases. In particular, a G-martingale will be a viscosity solution of a G-heat equation with path dependent terminal condition. As in the viscosity theory of standard PDEs, we shall prove the existence, stability, and uniqueness of viscosity solutions. The crucial step is to prove the comparison principle. In standard theory, the arguments rely heavily on the compactness of the state space, which is not true in the path dependent case. We decompose the problem into a partial comparison principle and a variation of the Peron's method. Such