Simulation of BSDEs and Wiener chaos expansions

Philippe Briand   Céline Labart

Université de Savoie, LAMA UMR 5127, 73376 Le Bourget du Lac Cedex, FRANCE

philippe.briand@univ-savoie.fr   celine.labart@univ-savoie.fr

Tis talk is based on a joint work with Céline Labart. We are interested in this paper in the numerical simulation of solutions to Backward Stochastic Differential Equations. There are several existing methods to handle this problem and one of the main difficulty is to compute conditional expectations.

Even though our approach can also be applied in the case of the dynamic programmation equation, our starting point is the use of Picard's iterations

$$Y^{q+1} = \xi + \int_t^T f\left(Y_s^q, Z_s^q\right) ds - \int_t^T Z_s^q \cdot dB_s, \quad 0 \leq t \leq T,$$

and to write this equation in a forward way

$$\begin{aligned} Y_t^{q+1} &= \mathbb{E}\left(\xi + \int_0^T f\left(Y_s^q, Z_s^q\right) ds \,\middle|\, \mathcal{F}_t\right) - \int_0^t f\left(Y_s^q, Z_s^q\right) ds, \\ Z_t^{q+1} &= D_t Y_t^{q+1} = \lim_{s \uparrow t} D_s Y_t^{q+1}. \end{aligned}$$

In order to compute the previous conditional expectation, we use a Wiener Chaos expansion of the random variable

$$\xi + \int_0^T f\left(Y_s^q, Z_s^q\right) ds.$$

Of course, from a practical point of view, we keep only a finite number of terms in this expansion.

One of the key point in using such an decomposition is the fact that there exist explicit formulas for both

$$\mathbb{E}\left(\xi + \int_0^T f\left(Y_s^q, Z_s^q\right) ds \,\middle|\, \mathcal{F}_t\right), \quad \text{and} \quad Z_t^{q+1} = D_t Y_t^{q+1}.$$

Using these formulas and starting from M trajectories of the underlying Brownian motion we are able to construct M trajectories of the solution (Y, Z) to the BSDE.

We will present numerical experiments and preliminary results on the error analysis.