Topics in Computational Finance
a view from the trenches

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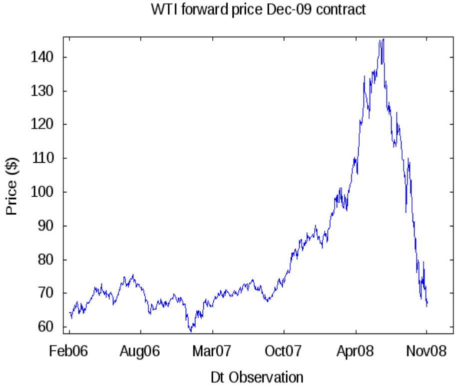
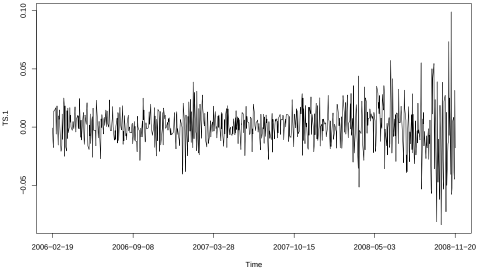
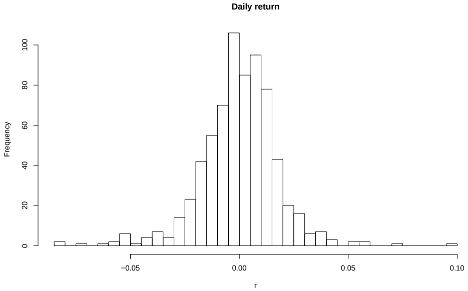
Closing price of December 2009 WTI contract.



Time Series of Daily Return



Histogram of Daily Return



Fitting a density to observed returns

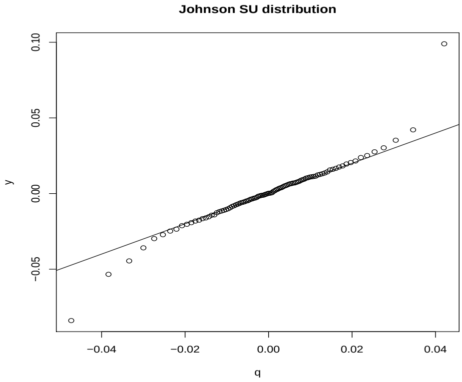
The Johnson family of distributions. X : observed Z : $N(0,1)$

$$Z = \gamma + \delta \ln(g(\frac{x-\xi}{\lambda})) \tag{1}$$

where :

$$g(u) = \begin{cases} u & SL \\ u + \sqrt{1+u^2} & SU \\ \frac{u}{1-u} & SB \\ e^u & SN \end{cases}$$

Johnson SU Distribution - December 2009 WTI contract.



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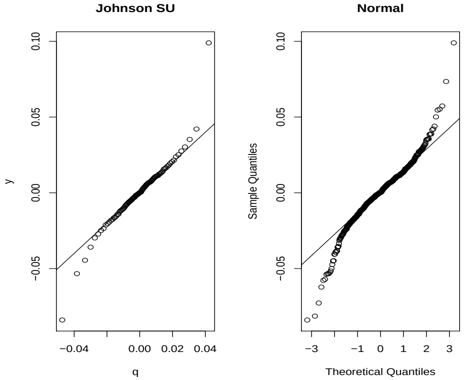
Johnson SU vs. Normal



The Generalized Lambda Distribution

Tukey's Lambda distribution :

$$Q(u) = \lambda + \frac{u^{\lambda} - (1-u)^{\lambda}}{\lambda} \tag{2}$$

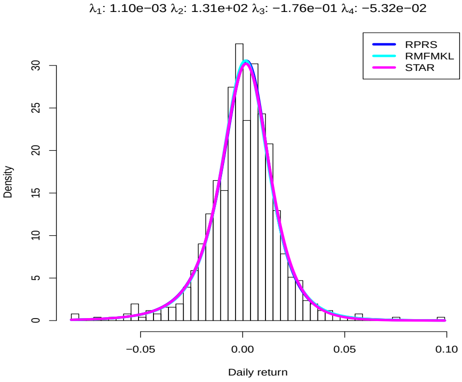
Generalized Lambda distribution :

$$Q(u) = \lambda_1 + \frac{u^{\lambda_3} - (1-u)^{\lambda_4}}{\lambda_2} \tag{3}$$

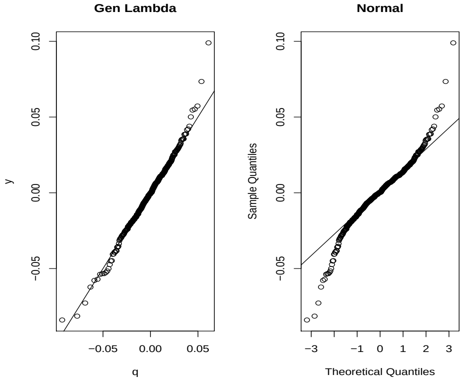
where :

$$Pr(X < Q(u)) = u$$

Density of daily return fitted with Generalized Lambda density



Gen. Lambda vs. Normal



Volatility Clustering

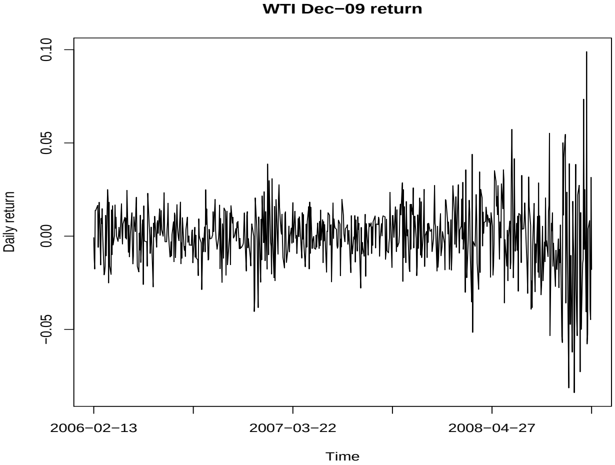


FIGURE: Series of daily returns

Autocorrelation of return

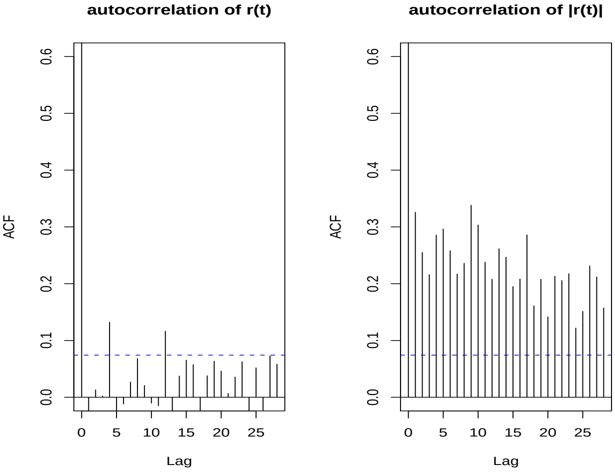


FIGURE: ACF of r_t and $|r_t|$

Summary - Price Process

- No evidence of linear autocorrelation of return
- Large excess kurtosis, incompatible with normal density
- Distribution of return is well approximated by a Johnson SU, to a lesser extend by a Generalized Lambda distribution
- Observable autocorrelation of $|r_t|$ and r_t^2, suggesting autocorrelation in the volatility of return.

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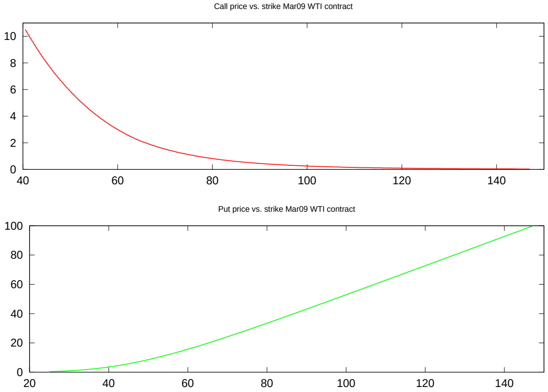
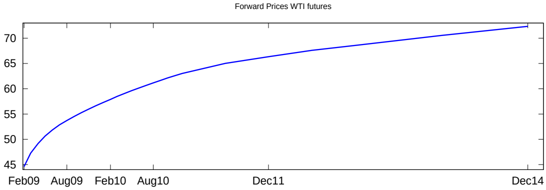
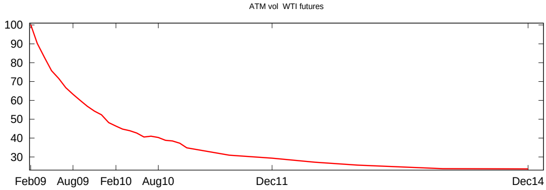
Quoted option prices - March 2009 WTI contract

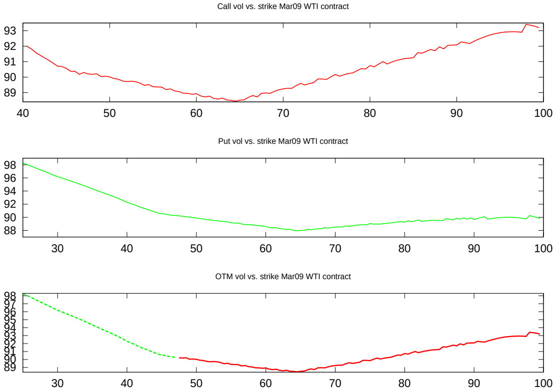


Volatility term structure - NYMEX WTI options

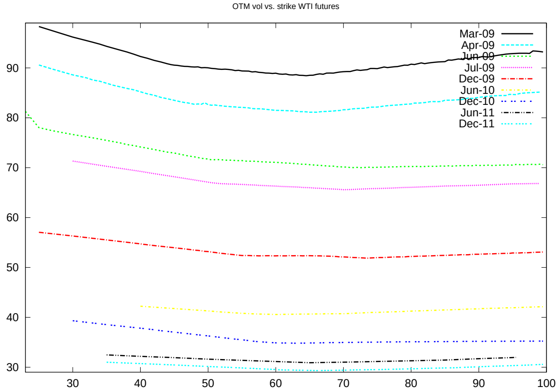




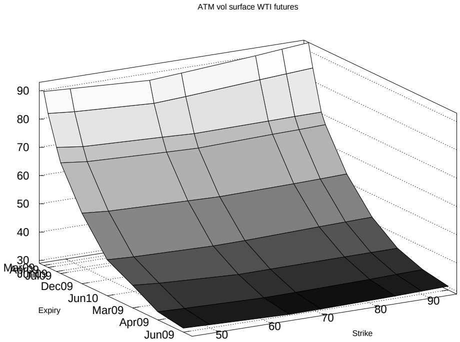
Implied Volatility - March 2009 WTI contract



Implied Volatility Cross-sections - NYMEX WTI options



Implied Volatility Surface - NYMEX WTI options



Summary - Volatility Surface

- Mean-reverting process for volatility
- Smile slope decreases as a function of $\frac{1}{\sqrt{T}}$
- Smile convexity decreases as a function of $\frac{1}{T}$
- Assymetry between call and put smile

The Perfect Model

- Multi-factor to capture the dynamic of the term structure
- Returns with fat tails : GL, VG, stochastic volatility
- Jumps (with up/down assymetry)
- mean reverting stochastic volatility for volatiity clustering

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The One and Only Commandment of Quantitative Finance

> *If you want to know the value of a security, use the price of another security that's as similar to it as possible. All the rest is modeling.*

Emanuel Derman, "The Boy's Guide to Pricing and Hedging"

Valuation by Replication

Replication can be :

- static : useful even if only partial
- dynamic : model are needed to describe possible outcome

Objective :

- To minimize the impact of modeling assumptions.
- Holy Grail : A model-free dynamic hedge

Conclusion

- Pricing and hedging by replication
- Mesure of market risk :
  - Not in terms of model parameters
  - but in terms of simple hedge instruments
- The reasons for the longevity of Black-Scholes
  - “The wrong volatility in the wrong model to obtain the right price”
  - Black-Scholes as a formula to be solved for volatility : a normalization of price.
- Choose the model in function of the payoff pattern.

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Calibration Isues

- Market data is insufficient and of poor quality
- Model estimation is an ill-posed problem

Option data : Settlement prices of options on the Feb09 futures contract

NEW YORK MERCANTILE EXCHANGE
NYMEX OPTIONS CONTRACT LISTING FOR 12/29/2008

--------CONTRACT--------				TODAY'S SETTLE	PREVIOUS SETTLE	ESTIMATED VOLUME	DAILY HIGH	DAILY LOW
LC	02 09	P	30.00	.53	.85	0	.00	.00
LC	02 09	P	35.00	1.58	2.28	0	.00	.00
LC	02 09	P	37.50	2.44	3.45	0	.00	.00
LC	02 09	C	40.00	3.65	2.61	10	.00	.00
LC	02 09	P	40.00	3.63	4.90	0	.00	.00
LC	02 09	P	42.00	4.78	6.23	0	.00	.00
LC	02 09	C	42.50	2.61	1.80	0	.00	.00
LC	02 09	C	43.00	2.43	1.66	0	.00	.00
LC	02 09	P	43.00	5.41	6.95	100	.00	.00

Calibration of term structure model

Let $F(t,T)$ be the value at time t of a futures contract expiring at T. Assume a two factor model for the dynamic of the futures prices :

$$\frac{dF(t,T)}{F(t,T)} = B(t,T)\sigma_S dW_S + (1 - B(t,T))\sigma_L dW_L$$

with

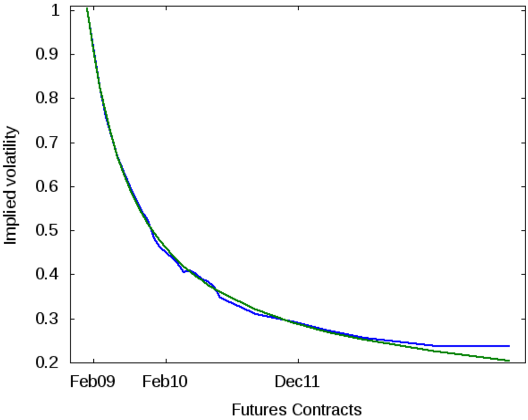
$$\begin{aligned} B(t,T) &= e^{-\beta(T-t)} \\ < dW_S, dW_L > &= \rho \end{aligned}$$

First approach : non-linear least-square on implied volatility

Given the implied volatility per futures contract $\widehat{\sigma(T_i)}$, find the parameters $\sigma_L, \sigma_S, \rho, \beta$ that solve :

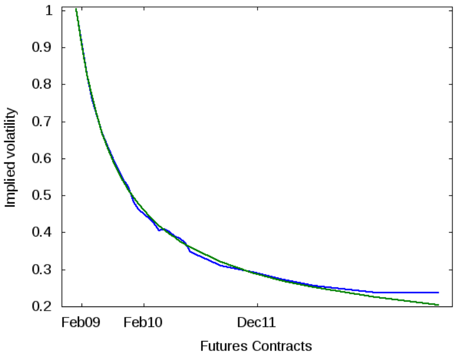
$$
\begin{array}{ll}
\min & \sum_{i=1}^{N} [\widehat{\sigma(T_i)} - \sqrt{V(T_i, \sigma_S, \sigma_L, \rho, \beta)}]^2 \\
\text{such that} & \\
 & \rho^- \leq \rho \leq \rho^+ \\
 & \sigma_L^- \leq \sigma_L \leq \sigma_L^+ \\
 & \sigma_S^- \leq \sigma_S \leq \sigma_S^+ \\
 & \beta^- \leq \beta \leq \beta^+
\end{array}
$$

Calibration results



Parameters :

$$\sigma_S = 1.07, \sigma_L = .05, \rho = 1.0, \beta = 2.57$$


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
Implied volatility
Feb09
Feb10
Dec11
Futures Contracts

Calibration results, varying ρ

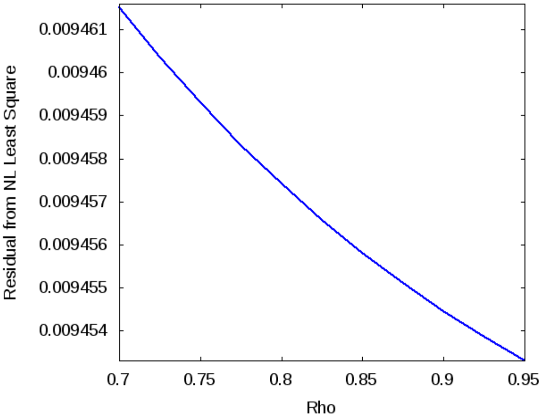
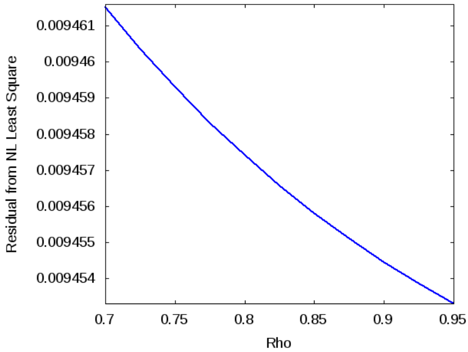


FIGURE: Mean error for various ρ fixed


Residual from NL Least Square
0.009461
0.00946
0.009459
0.009458
0.009457
0.009456
0.009455
0.009454
0.7
0.75
0.8
0.85
0.9
0.95
Rho

Optimal parameters, ρ fixed

ρ	mean error	σ_S	σ_L	β
0.70	0.0026	1.07	0.0603	2.5
0.72	0.0026	1.07	0.0598	2.51
0.75	0.00259	1.07	0.0593	2.52
0.77	0.00259	1.07	0.0588	2.52
0.80	0.00259	1.07	0.0583	2.53
0.82	0.00259	1.07	0.0578	2.54
0.85	0.00259	1.07	0.0573	2.54
0.88	0.00259	1.07	0.0568	2.55
0.90	0.00259	1.07	0.0564	2.55
0.92	0.00259	1.07	0.0559	2.56
0.95	0.00259	1.07	0.0554	2.57

Parameter relationships to historical data

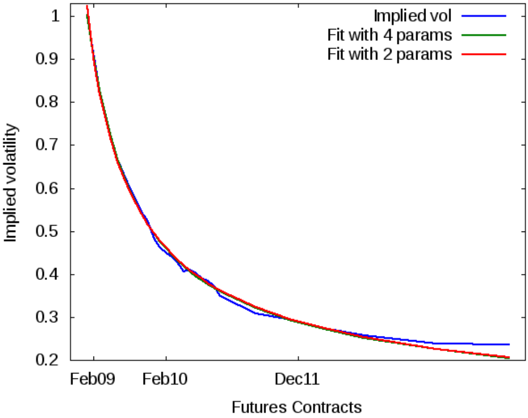
$$\frac{dF(t,T)}{F(t,T)} = B(t,T)\sigma_S dW_S + (1 - B(t,T))\sigma_L dW_L$$

$$\frac{dF(t,T)}{F(t,T)} \to \sigma_L dW_L, \quad T \to \infty$$

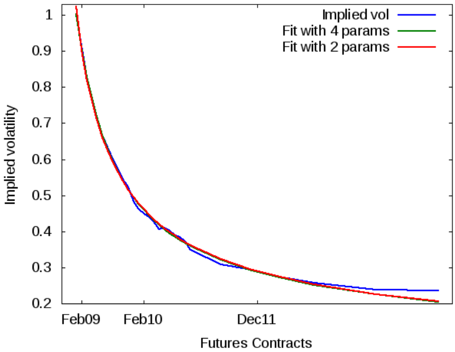
$$\frac{dF(t,T)}{F(t,T)} \to \sigma_S dW_S, \quad T \to 0$$

$$\rho \approx < \frac{dF(t,T_\infty)}{F(t,T_\infty)}, \frac{dF(t,T_0)}{F(t,T_0)} >$$

Hybrid Calibration - Version 1



Estimate ρ and σ_L historically ($\rho = .87$, $\sigma_L = .12$), calibrate σ_S and β to implied ATM volatility.


Implied vol
Fit with 4 params
Fit with 2 params
Implied volatility
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
Feb09
Feb10
Dec11
Futures Contracts

Hybrid Calibration - Version 2

Estimate ρ and σ_L historically, calibrate all parameters to implied ATM volatility, with a penalty on ρ and σ_L for deviation from historical values.

New objective function :

$$\min \sum_{i=1}^{N} [\widehat{\sigma(T_i)} - \sqrt{V(T_i, \sigma_S, \sigma_L, \rho, \beta)}]^2 + w_\rho \phi(\rho - \overline{\rho}) + w_{\sigma_L} \phi(\sigma_L - \overline{\sigma_L})$$

Penalty functions :

$$\phi(x) = x^2$$

$$\phi(x) = \begin{cases} 0 & \text{if } |x| < \epsilon \\ (|x| - \epsilon)^2 & \text{otherwise} \end{cases}$$

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Division of Labour

Models used to price and hedge the portfolio of a typical exotic derivatives desk :

	Nb Trades	Model	Reprice	EOD
Hedge	10^6	BS	< 1 min	< 10 min
Exotic Assets	10^3	Monte Carlo + StoVol, Local Vol, Jumps etc.	< 30 min	2-6 hours

Practical Advantages of MC pricing

- Flexibility (with pay-off language)
- Consistent pricing of Exotics and Hedge
- Easy to switch model to assess model risk
- Only feasible solution for large dimension risk models

Open research topics :

- Stability of Greeks in MC framework
- Robust variance reduction methods
- Modeling the contract rather than the payoff

Conclusion

- With a model comes model risk : minimize this risk by :
  - looking first for replicating instruments (even partial)
  - using a model to price
    - the residual payoff
    - the non-standard payoffs
  - Express risk in terms of simple hedge instruments rather than model risk factors
- MC simulation is the workhorse of exotic pricing, but the method suffers from many practical limitations.