Stochastic variational inequalities with oblique subgradients
by
Aurel Răşcanu

The lecture is based on a joint paper with Anouar Gassous, my Ph.D. student in ITN programme. We study the stochastic variational inequalities with oblique subgradients:

$$\begin{cases} dX_t + H(X_t)\,\partial\varphi(X_t)(dt) \ni f(t, X_t)\,dt + g(t, X_t)\,dB_t, & t > 0 \\ X_0 = \xi, \end{cases}$$

where $\varphi : \mathbb{R}^d \to]-\infty, +\infty]$ is a lower semicontinuous convex function, $\partial\varphi$ denote the subdifferential of φ

$$\partial\varphi(x) \overset{def}{=} \left\{\hat{x} \in \mathbb{R}^d : \langle \hat{x}, y - x\rangle + \varphi(x) \le \varphi(y)\,,\ \text{for all } y \in \mathbb{R}^d\right\}.$$

and $H = (h_{i,j})_{d\times d} \in C_b^2\left(\mathbb{R}^d; \mathbb{R}^{2d}\right)$. The vector $H(x)\,\hat{x}$ with $\hat{x} \in \partial\varphi(x)$ is called *oblique subgradient*. In our approach we start with a generalized convex Skorohod problem and by some continuity and tightness properties the existence in stochastic case is obtained. The very big difficulty is to control the term $H(x)\,\partial\varphi(x)$ which is not monotone or Lipschitz continuous.