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Backward stochastic partial differential equations driven by infinite dimensional martingales and applications

Abstract: This talk addresses on a complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ a result of the existence and uniqueness of the solution to a backward stochastic partial differential equation of the following type:

$$(BSPDE) \begin{cases} -dY(t) = (\, A(t)\, Y(t) + F(t, Y(t), Z(t)\mathcal{Q}^{1/2}(t))\,)\, dt - Z(t)\, dM(t) - dN(t), \\ Y(T) = \xi. \end{cases}$$

Here ξ is given as the terminal value, $A(t,\omega)$ is a predictable linear operator on a separable Hilbert space H, M is a continuous martingale in H and $\mathcal{Q}$ is its local covariation operator.

Assuming that $A(t,\omega)$ is coercive for a.a. $(t,\omega) \in [0,T] \times \Omega$ and $F : [0,T] \times \Omega \times H \times L_2(H) \to H$ satisfies a global Lipschitz condition, we show that this BSPDE admits a unique solution (Y, Z, N) of predictable processes taking values in $V \times L_2(H) \times \mathcal{M}^{2,c}(H)$ such that $N(0) = 0$ and N is very strongly orthogonal to M in the sense of Métivier [3], where $V \subset H$. This space $\mathcal{M}^{2,c}(H)$ consists of square integrable continuous martingales which take values in H, while $L_2(H)$ denotes the space of Hilbert-Schmidt operators from H into itself.

We apply this result for instance in studying the maximum principle for a controlled stochastic evolution system as in [2]. The references [1] and [4] are related to this work.

References

[1] Al-Hussein, A.: Backward stochastic partial differential equations driven by infinite dimensional martingales and applications. *Warwick-2009 preprint, submitted.*, (2009).

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[3] Métivier, M.: Semimartingales. A course on stochastic processes, *de Gruyter Studies in Mathematics*, **2**, Walter de Gruyter & Co, Berlin-New York. (1982).

[4] Øksendal, B.; Proske, F.; Zhang, T.: Backward stochastic partial differential equations with jumps and application to optimal control of random jump fields. *Stochastics*, **77**, no. 5, 381–399, (2005).