Conformal geometry, harmonic maps and biharmonic maps

Hajime URAKAWA

Tohoku University

Let (M, g), (N, h) be two Riemannian manifolds. In 1991, J. Eells and M.J. Ferreira gave a theorem representing each homotopy class $\mathcal{H}$ of of M into N by a harmonic map from (M, fg) into (N, h), if we choose a C^∞ function $f > 0$ on M. Our problem is to represent each homotopy class of M into N by a proper biharmonic map, i.e., biharmonic map, but not harmonic map from (M, fg) into (N, h). Then, (1) I will give a formula on conformal change of biharmonic energy tension field, and also energy tension field. (2) I will give the ODE about $f > 0$ for which the identity map of $(\mathbb{R}^m, fg)$ into $(\mathbb{R}^m, g)$ is a proper biharmonic map, where $(\mathbb{R}^m, g)$ is the standard Euclidean space. (3) Finally, I will give an exact solution f such that the identity map of $(\mathbb{R}^4, fg)$ into $(\mathbb{R}^4, g)$ is a proper biharmonic map.