Schouten curvature functions on conformally flat manifolds

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Notation

Conn. Riem. mfd. (M, g), dim. $n \geq 3$, Ric Ricci, κ normed scalar curvature

Schouten tensor and Schouten curvature

$S := \frac{1}{n-2}(Ric - \frac{n}{2}\ \kappa \cdot g)$ Schouten tensor
the g-associated operator $\mathcal{S}$ is selfadjoint and has eigenvalues k_i; their elementary symmetric function $\sigma_j(S)$ of order j is the *Schouten curvature function of order* j:

Theorem 1.
(M^n, g) compact loc. conf. flat mfd., $\sigma_k(S) = const > 0$ for some $k \in \{2, \cdots, n\}$; if S is semi-pos. def., then (M^n, g) is a space form of pos. sect. curvature.

Theorem 2.
(M^n, g) compact loc. conf. flat mfd., $\sigma_2(S_g) = const > 0$ and $Ric \geq 0$; then:
$\mathbf{n = 3}$: *(M^n, g) is a space form*
$\mathbf{n \geq 4}$: *(M^n, g) is either a space form or is $\mathbb{S}^1 \times N^{n-1}$ with N a space form.*

Theorem 3 (Isometric Classification)
(M^n, g) complete loc. conf. flat mfd., $\kappa = const$, $Ric \geq 0$ then the univ. cover $(\tilde{M}^n, \tilde{g})$ of (M^n, g) is isometric to $\mathbb{S}^n(c)$, $\mathbb{R}^n$ or $\mathbb{R} \times \mathbb{S}^{n-1}(c)$.

Wew give applications to complete centroaffine Tchebychev hypersurfaces. They are affine spheres or conformally flat.