Polyharmonic maps in the critical dimension

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Polyharmonic maps $u : M \to N$ between Riemannian manifolds can be defined as critical points of the m-polyenergy $E_m(u) := \int_M |D^m u|^2$. Depending on whether we read $D^m u$ as the covariant derivative with respect to N or as the full derivative with respect to some Euclidean space in which N is embedded, we speak of "intrinsically" or "extrinsically" polyharmonic maps. They satisfy a higher order elliptic PDE system with critical nonlinearities. In the special case $M = \mathbb{R}^{2m} \to N$, the polyenergy E_m coincides with a conformally invariant "Paneitz energy" $\int_M \langle P_{2m} u, u\rangle$ where P_{2m} denotes the $2m$-th order Paneitz-type operator.

We establish smoothness (away from the boundary) of weakly polyharmonic maps $\mathbb{R}^{2m} \supseteq \Omega \to N$ in the critical dimension $2m$ (joint work with Christoph Scheven). Moreover, for the extrinsic case and $\dim M = 2m$, the heat flow admits eternal solutions with finitely many singular times.